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LOWER ESTIMATE OF THE FLIGHT RANGE OF A FIRE-EXTINGUISHING LIQUID DROP

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A closed analytical solution of a nonlinear differential equation with variable coefficients which describes the vertical motion of an ascending evaporating drop as a material point of variable mass has been constructed in Airy functions. It is shown that this solution yields a lower estimate of the flight range of the drop when it outflows at an arbitrary angle to the horizon.

In delivering a fire-extinguishing liquid in the form of a sprayed jet for a long distance, a considerable portion of the liquid is scattered in flight and does not reach the combustion site [1]; therefore, calculation of the limiting range of effective delivery of a sprayed liquid is a very urgent problem whose solution will make it possible to decrease losses of a fire-extinguishing substance, i.e., to improve the effectiveness of its use.

Problems on the ballistics of individual drops as particles of a sprayed jet were considered in [2–6]. It was assumed that a drop had a spherical shape and that its radius decreased due to evaporation by the linear law in time. The force of aerodynamic resistance to the motion was considered proportional to the squared velocity of gas flow around the drop. The problem of ballistics in such a statement is reduced to solution of a system of nonlinear differential equations with variable coefficients, which requires the application of numerical methods. The construction of closed analytical solutions, just as simple computational formulas, presents difficulties. Approximate analytical solutions of the nonlinear Cauchy problem were successfully obtained in [4–6], provided the influence of the gravity force on the bending of the trajectory of the flight of a drop was neglected.

We suggest a different approach to determining the effective range of flight of a drop. It consists of estimating this range from below (in the sense that the theoretical range given by the numerical solution of the nonsimplified Cauchy problem will always exceed the result of the proposed estimate). In order to obtain an analytical estimate, one does not need to solve a complex nonlinear system; it suffices to find the solution of one differential equation which describes the vertical motion of a drop upward. In this case, the weight of the drop is opposite to the direction of its motion, i.e., it exerts a maximum retarding action. As a result, in a fixed interval of time, while ascending vertically, the drop covers a smaller distance than at other angles of efflux to the horizon.

Just as in [2, 3], we assume that the force of aerodynamic resistance to the motion of a liquid particle is proportional to the squared velocity of gas flow around it. The decrease in the current radius of the drop $r(t)$ in time t due to evaporation is described by the expression [7–9]

$$r(t) = r_0 \sqrt{1 - \varepsilon t} \,. \tag{1}$$

Here $r_0 = r(0)$; ε^{-1} is the time of complete evaporation of the drop. Recommendations for calculating ε under specific conditions of the flight of a drop can be found in [9].

Within the framework of the assumptions made, a change in the flight velocity $v = v(t)$ during vertical ascending of the drop is described by the equation

$$\frac{dv}{dt} + \frac{\beta v^2}{r_0 \sqrt{1 - \varepsilon t}} = - g \,, \tag{2}$$

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which is supplemented with the initial condition

$$v(0) = v_0 \tag{3}$$

with the velocity of vertical efflux of the drop being denoted by v_0.

Taking into account (1), we will introduce a new variable:

$$\xi = \sqrt{1 - \varepsilon t}\,, \quad \frac{d\xi}{dt} = -\frac{\varepsilon}{2\xi}\,, \quad \frac{d}{dt} = -\frac{\varepsilon}{2\xi}\frac{d}{d\xi}\,.$$

Then the sought-for velocity will be defined by the equation

$$\frac{dv}{d\xi} - bv^2 = \omega\xi\,, \tag{4}$$

where $b = 2\beta(\varepsilon r_0)^{-1}$; $\omega = 2g\varepsilon^{-1}$.

To go over from a nonlinear problem to a linear one we will express v via the auxiliary function w and its derivative. Assuming that

$$v = -\frac{1}{bw}\frac{dw}{d\xi}\,, \tag{5}$$

instead of Eq. (4) we obtain

$$\frac{d^2w}{d\xi^2} + a^3\xi w = 0\,. \tag{6}$$

Here $a = \sqrt[3]{2bg\varepsilon^{-1}}$. The general solution of (6) is the sum

$$w = c_1 Ai(-a\xi) + c_2 Bi(-a\xi)\,, \tag{7}$$

in which c_1 and c_2 are arbitrary constants; $Ai(-\eta)$ and $Bi(-\eta)$ are the Airy functions [10, 11].

Taking into account expressions (5) and (7), we arrive at the general solution of Eq. (2):

$$v(t) = \frac{a}{b}\frac{cAi'(-\eta) + Bi'(-\eta)}{cAi(-\eta) + Bi(-\eta)}\,. \tag{8}$$

Here $c = c_1 c_2^{-1}$ is an arbitrary constant; $\eta = a\sqrt{1 - \varepsilon t}$; $Ai'(-\eta)$ and $Bi'(-\eta)$ are the derivatives of the Airy functions with respect to η. Their values, just as those of the Airy functions themselves, are available in the tables published in [10, 11] and in other publications on special functions. We find the constant c with allowance for the initial condition (3):

$$c = \frac{ab^{-1}Bi'(-a) - v_0 Bi(-a)}{v_0 Ai(-a) - ab^{-1}Ai'(-a)}\,. \tag{9}$$

Thus, the lower estimate of the absolute value of the drop flight velocity at any instant of time $t \in [0; \varepsilon^{-1})$ can be conventionally obtained with the aid of analytical solutions (8), (9) and tables of special functions [10, 11]. It is more complex to estimate the range of the drop flight. This is associated with the computation of the integral

$$z(t) = \int_0^t v(t)\,dt\,. \tag{10}$$

It is not reduced to tabulated functions and must be calculated numerically on a computer.

TABLE 1. Results of Numerical Integration of the System of Differential Equations of Motion and the Lower Estimate of the Absolute Value of Velocity

t, sec	$\dot{x}(t)$, m/sec	$\dot{y}(t)$, m/sec	$v_{\rm n}(t)$, m/sec	$v(t)$, m/sec
0.02	96.65	35.00	102.79	102.69
0.05	72.89	26.14	77.44	77.23
0.10	50.63	17.74	53.65	53.31
0.15	37.80	12.82	39.91	39.47
0.20	29.31	9.50	30.81	30.28
0.25	23.05	7.04	24.10	23.50

TABLE 2. Computed Projections of the Motion of a Drop on the Trajectory, as Well as the Lengths of the Radius-Vector and the Lower Estimate of the Radius-Vector Lengths

t, sec	$x(t)$, m	$y(t)$, m	$R(t)$, m	$z(t)$, m
0.02	2.17	0.79	2.31	2.31
0.05	4.68	1.69	4.98	4.97
0.10	7.71	2.77	8.19	8.18
0.15	9.90	3.52	10.51	10.48
0.20	11.56	4.08	12.26	12.21
0.25	12.86	4.49	13.62	13.56

The use of a fire-extinguishing liquid is considered effective when the time of drop flight is limited to the value of $t_{\rm ad}$ at which the current radius of an evaporating particle is equal to half its initial radius [2, 3]. When the inequality $t \le t_{\rm ad} = 3(4\varepsilon)^{-1}$ is satisfied, the integral (10) can be estimated analytically, and the formula for its approximate value can be given.

In practical calculations, it turns out that $\eta < a < 1$. Therefore, taking into account the behavior of the Airy functions at small values of the argument [10], we introduce the asymptotics

$$v(t) \underset{g\to 0}{\to} v_{\rm a}(t) = \left[\frac{1}{v_0} + \frac{2\beta}{\varepsilon r_0}(1-\sqrt{1-\varepsilon t})\right]^{-1}, \tag{11}$$

the integral for which is expressed in terms of the elementary functions

$$S(t) = \int_0^t v_{\rm a}(t)\,dt = \frac{r_0}{\beta}\left(\sqrt{1-\varepsilon t} - 1 + A\ln\left(\frac{A-\sqrt{1-\varepsilon t}}{A-1}\right)\right) \tag{12}$$

with $A = 1 + \dfrac{\varepsilon r_0}{2\beta v_0}$ and $v_{\rm a}(0) = v(0)$.

Since over the integration interval $t < t_0 < 1$ sec with $v(t) \to v_{\rm a}(t)$ and $t \to 0$ the function

$$\Phi(t) = \int_0^t \left(v_{\rm a}(t) - v(t)\right)dt$$

is small in comparison with $S(t)$, it is positive and satisfies the inequality

$$\Phi(t) < t\left[v_{\rm a}(t) - v(t)\right] < S(t) - tv(t)\,.$$

In approximate calculations over the range of effective supply of fire-extinguishing substances $t \in [0; t_{ad})$, we may adopt that

$$\Phi(t) \approx \frac{1}{2} t \left[v_a(t) - v(t) \right].$$

This formula results if we use, approximately, a triangle instead of the figure formed by the vertical straight line and the curves of two monotonically decreasing functions $v(t)$ and $v_a(t)$ intersecting at $t = 0$.

Thus, for the lower estimate of the path covered by a drop on its escape at an arbitrary angle to the horizon we obtain the expression

$$z(t) \approx S(t) - \frac{1}{2} t \left[v_a(t) - v(t) \right]. \tag{13}$$

The calculation of $z(t)$ is reduced to application of Eqs. (8), (11)–(13) and tables of Airy functions. Computations were performed at $r_0 = 10^{-4}$ m, $\beta = 10^{-5}$, $\varepsilon = 3$ sec^{-1}, and $v_0 = 130$ m/sec. Corresponding to these is the value $t_{ad} = 0.25$ sec. In [12], a numerical integration in the system of two nonlinear differential equations with the above initial data for a drop outflowing at an angle $\theta_0 = 20^o$ to the horizon yielded the projections of the flight velocity $\dot{x}(t)$ and $\dot{y}(y)$, as well as the respective absolute values of the motion velocity $v_n = (\dot{x}^2 + \dot{y}^2)^{1/2}$ and the estimates of $v(t)$ which are given by Eqs. (8) and (9) (Table 1). A comparison shows that the estimate of the velocity from below is close to the absolute value of the velocity obtained by numerical integration of the system of differential equations in [12].

Table 2 present the values of $x(t)$ and $y(t)$, obtained numerically in [12], as well as the values of $R(t) = (x^2 + y^2)^{1/2}$ corresponding to them, and the estimates of $z(t)$ found with the aid of Eqs. (8), (11)–(13) are indicated. The values of $z(t) < R(t)$ and are close to the length of the radius-vector. The calculations confirm the effectiveness of the proposed estimate.

CONCLUSIONS

1. Within the framework of the assumptions adopted, the differential equation of the vertical motion of a drop has a closed solution in the Airy functions.

2. The formulas suggested are good for estimation of the range of flight of the evaporating drops of fire-extinguishing substances.

NOTATION

$Ai(-\eta)$, $Bi(-\eta)$, and $Ai'(-\eta)$, $Bi'(-\eta)$, Airy functions and their derivatives with respect to η; g, free fall acceleration; $R(t)$, radius-vector of motion on efflux at an angle of 20^o to the horizon; $r(t)$, r_0, current and initial radii of a drop; t, time; t_{ad}, time at which the current radius of a particle is half the initial one; $v_a(t)$, asymptotic value of the velocity of vertical rise; v_n, absolute value of the velocity on efflux at an angle 20^o; $v(t)$ and v_0, current and initial velocities of vertical rise of a drop; $x(t)$, $y(t)$, and $\dot{x}(t)$, $\dot{y}(t)$, coordinates of a drop on the trajectory and projection of the velocity of plane motion measured in a rectangular coordinate system $x0y$; $w(t)$, auxiliary function; $z(t)$, height of rise of a drop; β, reduced coefficient of aerodynamic resistance; ε, parameter that characterizes the intensity of evaporation. Subscripts: a, asymptotic; ad, admissible; n, numerical; 0, initial.

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